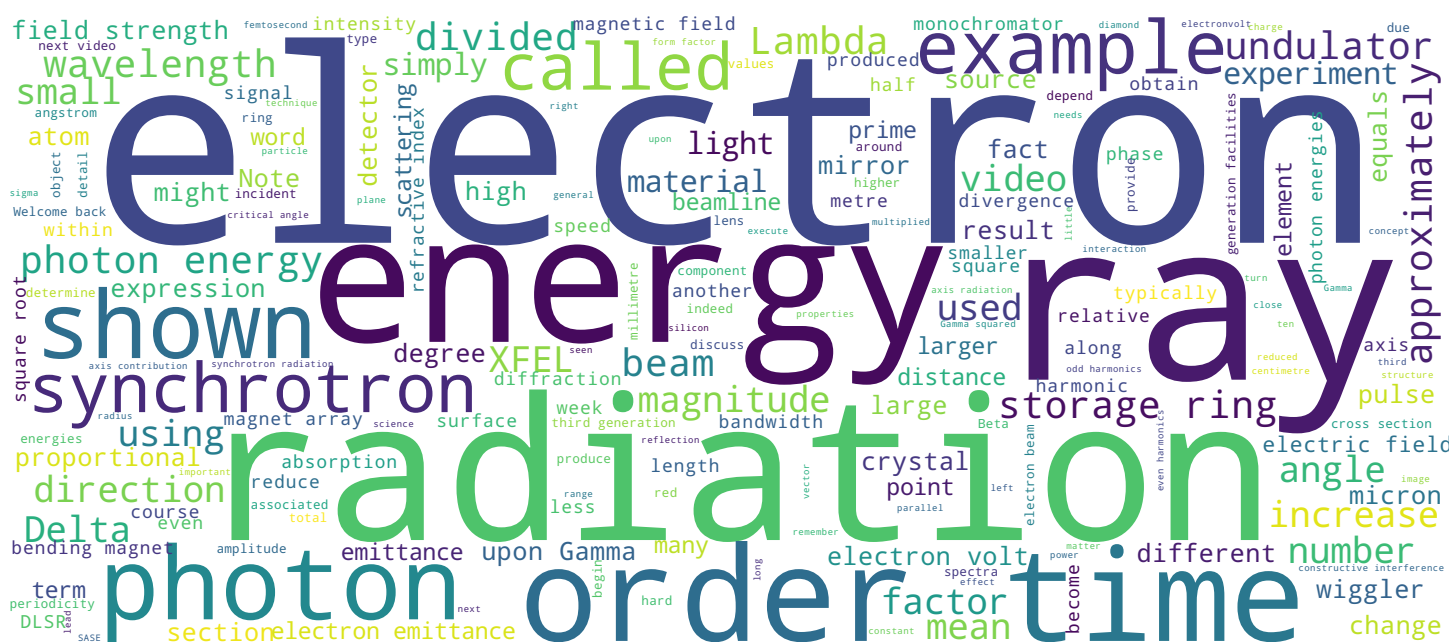


# Synchrotrons and x-ray free-electron lasers

## Techniques and applications

Prof. Philip Willmott



## Search MOOC



## Video



# Contents and objectives of this video



Welcome back. In this video, we'll take a closer look at undulators, the X-ray sources that define 3rd generation synchrotron facilities.

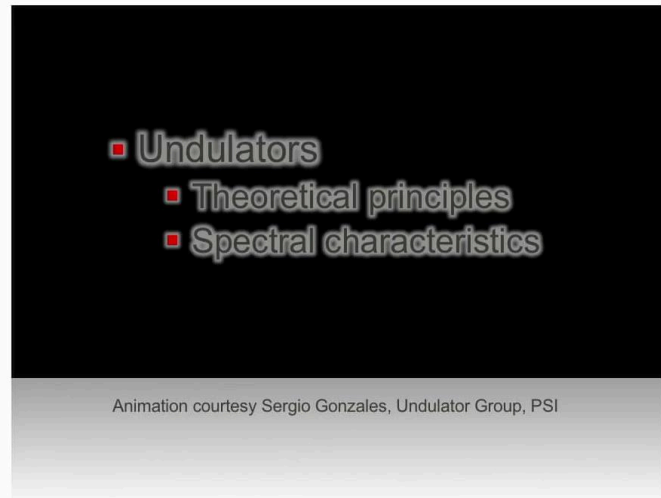
Notes

Summary



0m 05s

# Contents and objectives of this video



We will see that they differ from wigglers and that the radiation lobes emitted by the electrons as they execute their slalom path along the magnet array overlap and interfere with one another, resulting in spectra which are not the broadband affairs of bended magnets and wigglers, but instead consist of a regular set of sharp maxima spaced equally in energy.

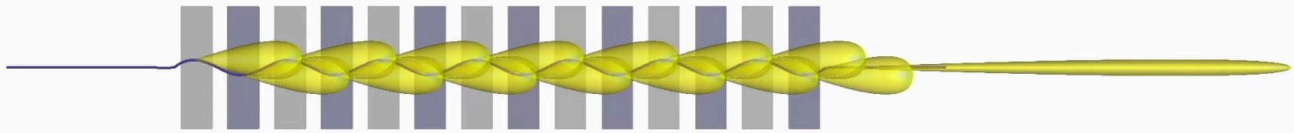
Notes

Summary



0m 27s

# From wiggler to undulator



Undulator,  $K \sim 1$

$$\sigma'^p = \frac{1}{\sqrt{mN}\gamma} = \sqrt{\frac{\lambda}{L}}$$

$m$  = harmonic

$N$  = # periods

Hard x-rays:  $\sigma'^p \sim 10 \mu\text{rad}$

As we have seen in the last video, wigglers have high  $K$  values, which means that loads of radiation are sufficient, laterally separated, that they can be considered as individual sources, as in a serial bank of bending magnets. A total intensity is thus proportional to the number of dipole pairs and is simply the sum of the individual intensities from each lobe. The resulting beam has a divergence in the horizontal plane equal to  $K$  upon  $\Gamma$ , where  $K$  is typically between 10 and 50. As  $K$  is reduced, the lobes will eventually overlap as a  $K$  value of unity is approached. The radiation from the lobes interfere and now it is the amplitudes of each lobe contribution, including their phases that must be vectorially added. The square of this vector sum determines the radiative intensity. The interference condition results in the selection of a series of narrow wavelength ranges, characteristic of undulator radiation. Undulator beams are much more collimated, the photon divergence in the horizontal plane is equal to 1 divided by  $\Gamma$ , divided by the square root of  $mN$ , where  $m$  is the harmonic number of the radiation and  $N$  is the number of periods. This turns out to be equal to the square root of  $\lambda$ , divided by  $L$ .

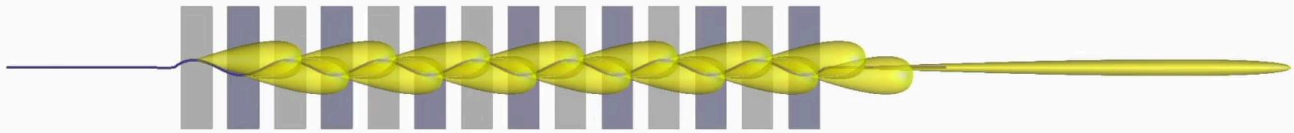
Notes

Summary



0m 42s

# From wiggler to undulator



Undulator,  $K \sim 1$

$$\sigma'^p = \frac{1}{\sqrt{mN\gamma}} = \sqrt{\frac{\lambda}{L}}$$

$m$  = harmonic

$N$  = # periods

Hard x-rays:  $\sigma'^p \sim 10 \mu\text{rad}$

where  $L$  is the length of the undulator. So for 1 angstrom radiation and a 2 meter undulator, the standard deviation divergence is approximately 7 micro radians. This should be compared to wiggler, which might have a divergence of five milli radians for  $K$  equals 25 due, primarily to the large electron oscillations induced by the wiggler magnet array.

Notes

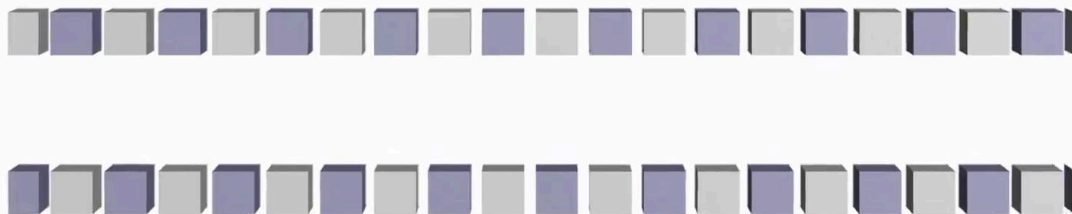
Summary



2m 17s

# How to reduce K?

$$K = 0.934 \lambda_{u,w} [\text{cm}] B_0 [\text{T}]$$



So how do we actually go about reducing K? We've already seen that K is proportional to the magnetic field strength and the ID periodicity. If we increase the gap between the upper and lower magnet arrays, we will reduce B zero. But this isn't an advisable approach as the intensity of the radiation drops off steeply with reduced field strength.

Notes

Summary



2m 46s



# How to reduce K?

$$K = 0.934 \lambda_{u,w} [\text{cm}] B_0 [\text{T}]$$



A better approach is to reduce the undulator periodicity. Because the field strength will also go down for a given magnetic material with smaller magnet size one also needs to reduce the gap between the upper and lower arrays. Undulators have peak magnetic field strength of the order of a Tesla and periodicity of a few centimetres.

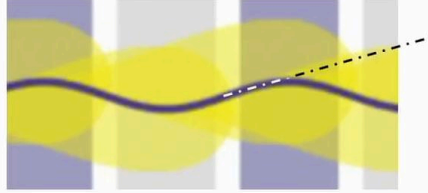
Notes

Summary



3m 14s

## K ~ 1? So what?



$$1/\gamma \sim \theta_{\max}$$

Emissions from each “undulation” overlap  
⇒ interference!!

Okay. We've now reduced the electrons oscillation amplitudes that yield maximum angular excursions, similar in magnitude to the natural opening angle,  $1/\gamma$  upon Gamma. This means that radiation emitted from individual undulations overlap and this leads to interference.

Notes

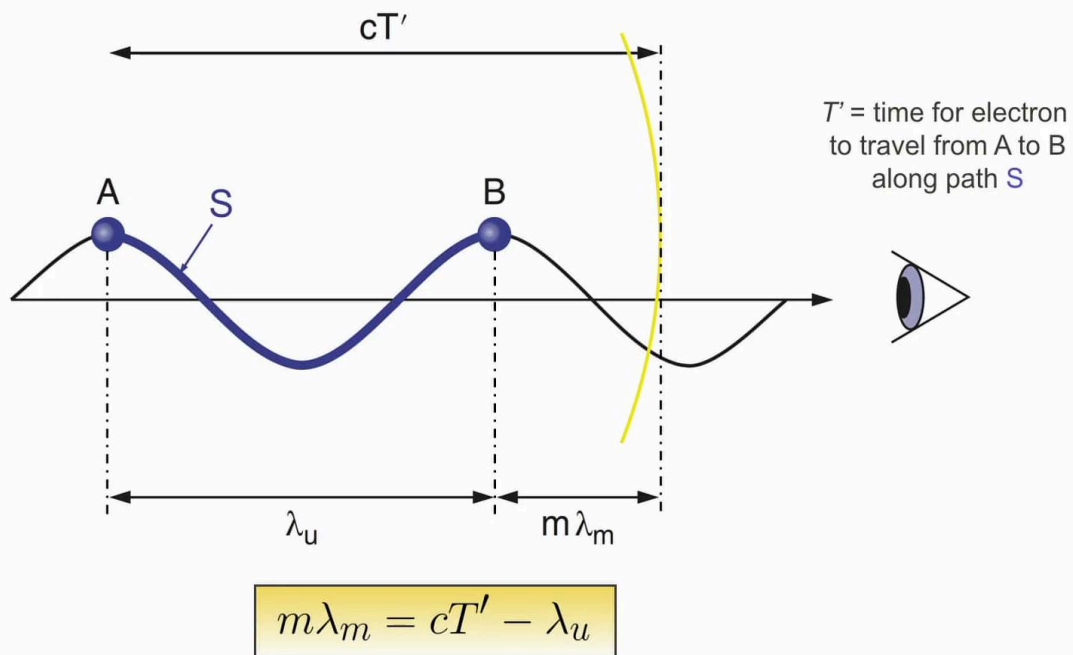
Summary



3m 40s



# Condition for constructive interference in undulators



So what are the conditions needed for constructive interference? This is met, if an electron emits radiation at point A and again at point B and these two radiation components differ by an integer number of wavelengths. Let the time for the electron to execute the single oscillation between A and B along the path S be  $C$  prime. In that time, the radiation emitted at A has travelled forward a distance  $C \times T$  prime. Therefore, the difference  $cT$  prime minus  $\lambda_u$  must equal an integer number of wavelengths and  $M$  times  $\lambda_m$ . The name of the game now is to determine  $T$  prime. Unfortunately, we can't approximate the path  $S$  to be simply equal to  $\lambda_u$  as the separation and  $M \lambda_m$  is very small. So the sinusoidal nature of  $S$  and its relative increase in length compared to a straight path plays a significant role, even if the amplitude is small.

Notes


Summary



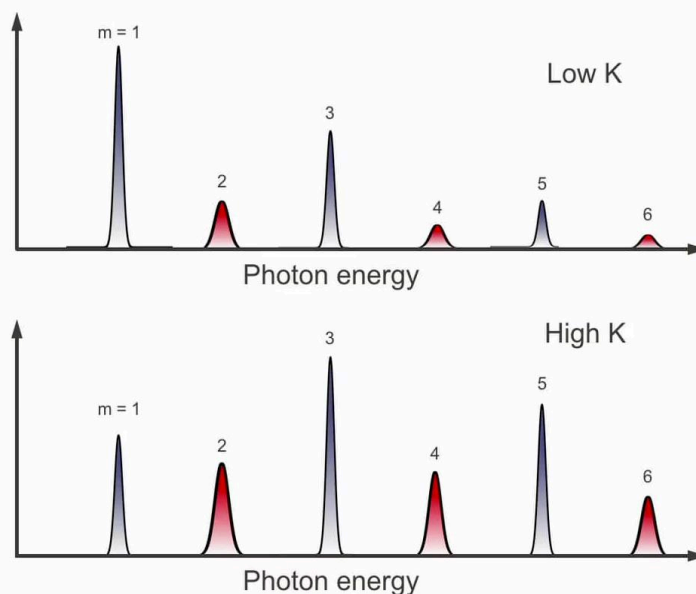
4m 05s

# Condition for constructive interference in undulators

$$m\lambda_m = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right)$$


  
 $\sim 0.02/5 \times 10^7$        $\sim 2$ 
  
 $\sim 10 \text{ \AA}$

See a detailed derivation of the equation describing constructive interference in an undulator in "Introduction to synchrotron radiation – techniques and applications", Philip Willmott, ed. 2 (2019): Section 3.9.3.



After some lengthy but straightforward maths that you're welcome to go through that first step in my accompanying book, it emerges that the condition for constructive interference occurs when integer multiples of the wavelength of radiation equal  $\lambda_u$  divided by  $2\gamma^2$  or multiplied by  $1 + K^2/2$ . The factor  $\lambda_u$  upon  $2\gamma^2$  comes from, on the one hand, the Doppler effect, which accounts for a factor of  $1/\gamma$  and on the other from special relativity and Lorentz length contraction, which provides a further factor of  $1/\gamma$ . For an undulator periodicity of about 2 centimetres, a  $\gamma$  of 5000, and a  $K$  value of 1.4 this results in  $m\lambda_m$  being of the order of 10 angstroms. As  $K$  changes by changing the undulator gap size the spectra change subtly. Low  $K$  spectra are dominated by low harmonics, and the even harmonics are significantly smaller than the odd harmonics. Higher  $K$  spectra extend to higher harmonics and the even harmonics are not as suppressed relative to the odd harmonics.

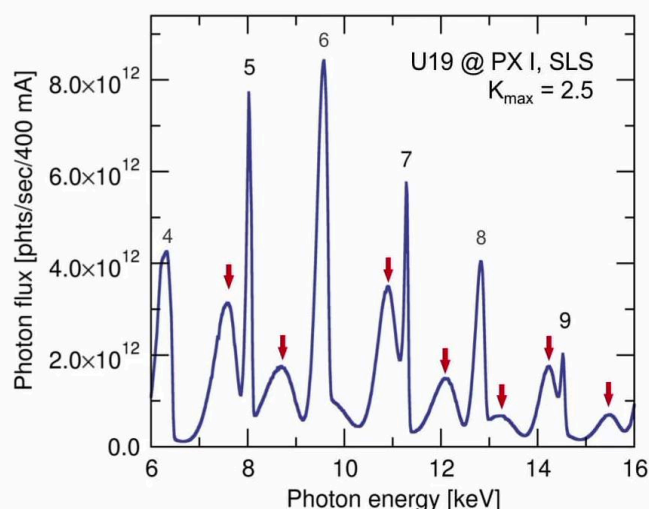
Notes

Summary

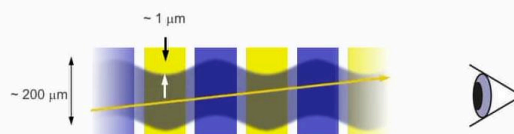


5m 20s

# Real undulator spectra @ third-generation facilities



$$m\lambda_m = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} + \gamma^2\theta^2 \right)$$



e.g.  $\theta = 1/2\gamma$ ,  $K = 1$

$$\frac{m\lambda_m|_{\theta=0}}{m\lambda_m|_{\theta=1/2\gamma}} = \frac{6}{7}$$

The real undulator spectra at 3rd generation facilities have additional features explained here for a relatively high K, U19 undulator at the PX I beamline at the Swiss light source. U19 indicates that the periodicity is 19 millimetres. First, the odd harmonics and broader even harmonics are easily identifiable between the fourth and ninth harmonics. The energy difference moving from harmonic to harmonic is approximately 1.62 KeV. In addition, there are even broader and relatively weak features highlighted with these red arrows. These are produced by off access radiation, originating from the fact that the electron beam has a finite width of the order of 100 microns. In fact, the oscillation amplitude induced by the undulators magnet array is significantly smaller by two orders of magnitude than the beams' width. An observer will see radiation from all across the river of electrons, as it were. The condition for constructive interference for off-axis radiation at an angle Theta to the central axis is given by the equation shown here, which differs from the on-axis equation by the term Gamma squared Theta squared highlighted here in red. Let's consider, for example, an angled Theta equals 1 upon 2, Gamma and K equals 1.

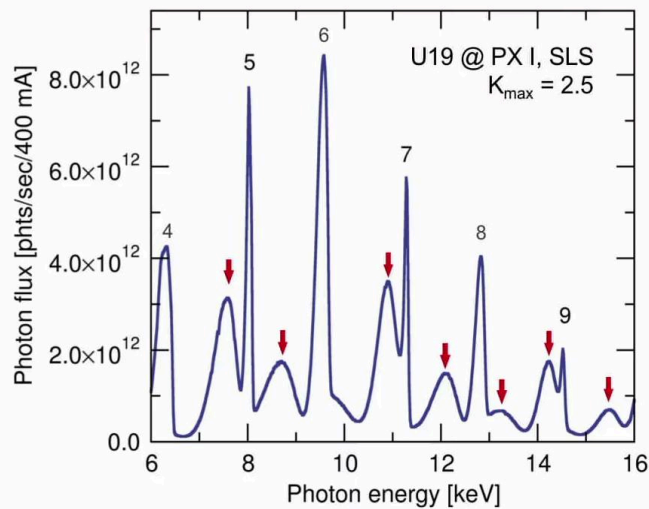
Notes

Summary

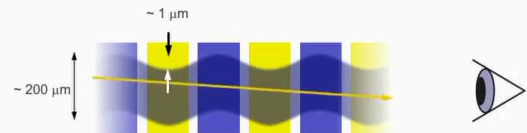


6m 46s

# Real undulator spectra @ third-generation facilities



$$m\lambda_m = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} + \gamma^2\theta^2 \right)$$



e.g.  $\theta = 1/2\gamma$ ,  $K = 1$

$$\frac{m\lambda_m|_{\theta=0}}{m\lambda_m|_{\theta=1/2\gamma}} = \frac{6}{7}$$

The term in the brackets is then equal to 1 plus a half plus a quarter. In other words, the off-axis contribution is 1/7 of the total, meaning that for the same harmonic number  $M$   $\lambda_m$  for the off-axis contribution is 7/6 that of the on-axis contribution. In terms of energy, the off-axis lobe is at 6/7 the energy of the axis value. We will see in the second section of this week's videos that undulator spectra at DLSR are much cleaner than at third generation facilities with little or no contribution from this off-axis radiation. This is due primarily to the fact that the wide river of the electrons that third generation facilities is maybe 10 times narrower at DLSR, meaning that off-axis radiation is largely suppressed.

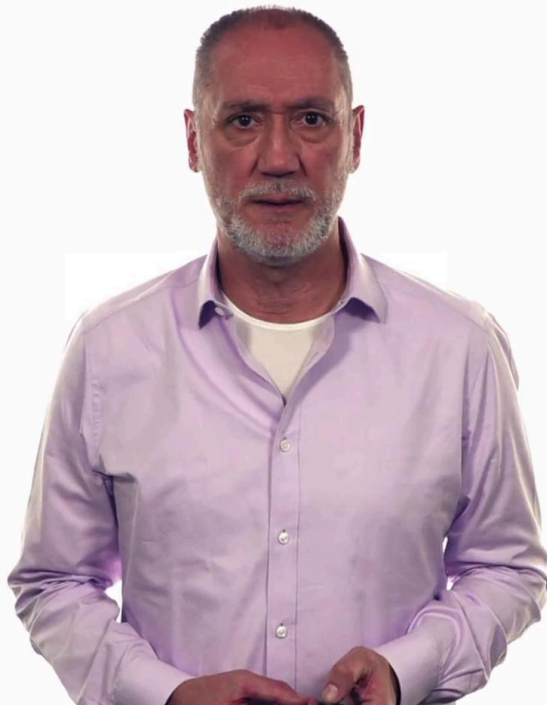
Notes

Summary



8m 32s

## In the next video...



Now, we've reviewed the basic features of undulator radiation we will see in the next video how to tune the spectra to deliver a specifically desired photon energy, including suppression of harmonic contamination and how one can change the polarisation of the radiation.

Notes

Summary



9m 31s